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Different scenarios leading to hyperchaos development in dynamical systems

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Introduction

Chaos is a typical attribute of nonlinear dynamical systems in various fields of science and technology. One of the conventional indicator of chaotic dynamics is the largest Lyapunov exponent. Chaos is implemented in a situation when in the spectrum of Lyapunov exponents for a flow there is one positive, one zero and at least one negative exponents. Using full spectrum of Lyapunov exponents it is possible to classify hyperchaos, when spectrum contains two or more positive Lyapunov exponents.

In the frame of this work we describe two scenarios leading to occurrence of hyperchaos. The first scenarios is a new scenario associated with appearance of Shilnikov's attractor, when saddle-focus with two-dimensional unstable manifold occurs via secondary Neimark-Sacker bifurcation and absorbs by chaotic attractor. For this scenario we will present cascade of secondary Neimark- Sacker bifurcations, corresponding to hierarchy of Shilnikov's attractors corresponding to hyperchaos. The second scenario was described previously, and associated with cascade of period doubling bifurcation of saddle-cycles with one-dimensional unstable manifold.

Object of study: 4D Rössler system

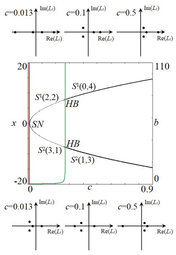
$$\dot{x} = -y - z,$$
$$\dot{y} = x + ay + w,$$
$$\dot{z} = b + xz,$$
$$\dot{w} = -cx + dw.$$

Equilibrium points

$$x_0 = \mp\sqrt{\frac{b(c-ad)}{d}}$$
$$y_0 = \mp\sqrt{\frac{bd}{c-ad}}$$
$$z_0 = \pm\sqrt{\frac{bd}{c-ad}}$$
$$w_0 = \pm c\sqrt{\frac{b}{d(c-ad)}}$$

Condition of saddle-node bifurcation

$$c = ad$$



Structure of parameter plane

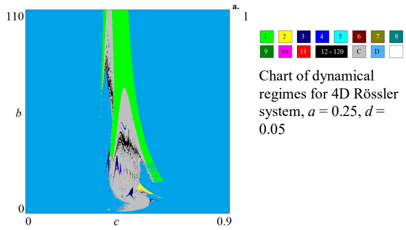


Chart of dynamical regimes for 4D Rössler system, $a = 0.25$, $d = 0.05$

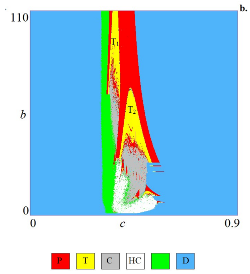
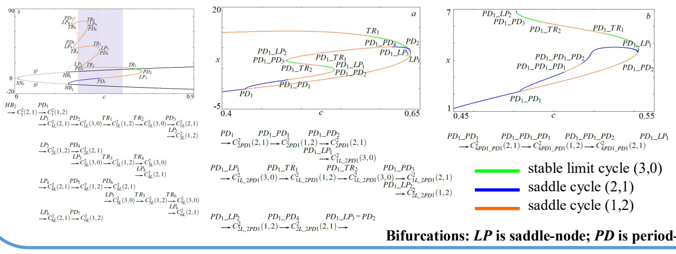


Chart of Lyapunov exponents for 4D Rössler system, $a = 0.25$, $d = 0.05$

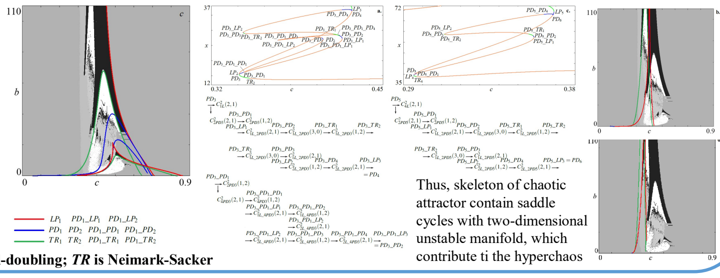
P is periodic oscillations;
T is quasiperiodic oscillations;
C is chaos;
HC is hyperchaos
D is divergency

Skeleton of chaotic attractor



stable limit cycle (3,0)
saddle cycle (2,1)
saddle cycle (1,2)

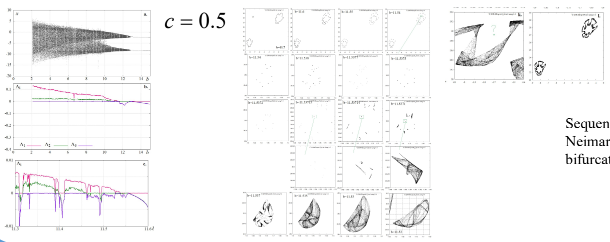
Bifurcations: *LP* is saddle-node; *PD* is period-doubling; *TR* is Neimark-Sacker



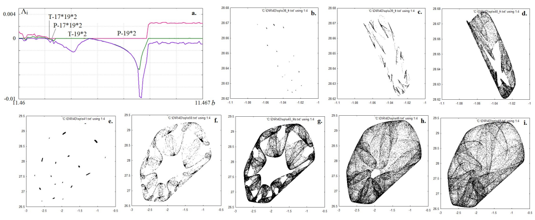
Thus, skeleton of chaotic attractor contain saddle cycles with two-dimensional unstable manifold, which contribute ti the hyperchaos

Hyperchaos occurrence via Shilnikov's homoclinic attractor (secondary Neimark-Sacker bifurcation)

$c = 0.5$

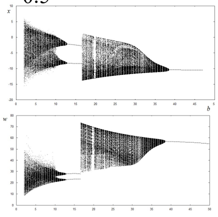


Sequence of secondary Neimark-Sacker bifurcation

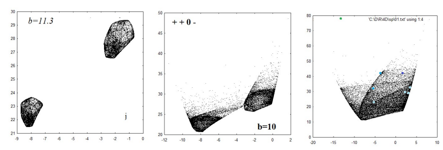


Hyperchaos occurrence via cascade of period-doubling bifurcations of saddles cycles

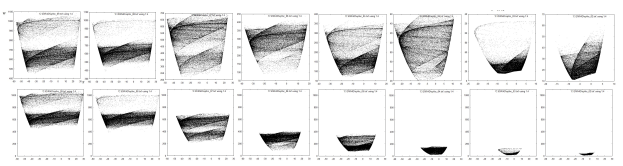
$c = 0.5$



Transformation of the two-component Shilnikov's attractor to hyperchaos with saddle cycles, occurred via cascad of period-doubling bifurcation of saddles (2,1)



Absorption of different saddle cycles at b=3



Conclusions

In the present work on the example of four-dimensional Rössler system two different scenarios are described. The skeleton of chaotic attractor is studied. The first scenario is associated with absorption of saddle-focus cycles occurred as a result of Neimark-Sacker bifurcation. We observe sequence of secondary Neimark-Sacker bifurcations and corresponding hierarchy of Shilnikov's homoclinoc attractors, which characterized by two positive Lyapunov exponents. Then we demonstrate transformation of Shilnikov's attractor to the hyperchaos absorbing saddle cycles (1,2) occurred through cascade of period-doubling bifurcations of the saddle cycles (2,1)

Reference

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